Technical Articles

Future Value Calculations and the Geometrically Varying Annuity

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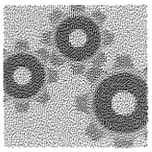

A fundamental tool of financial planning is the future value calculation. Whenever an advisor talks with a client about meeting a retirement goal, the discussion most certainly revolves around future value issues. When a planner discusses education planning, the focus of the discussion involves calculating a future value. Future value calculations are also used in insurance, investment planning and estate reviews. In short, almost every facet of the financial services profession deals with future value calculations.

It wasn't that long ago when calculating a future value was a tedious task. Before the widespread use of financial planning software and computer programs like Excel, planners who wanted to know the future value of savings or assets had to use a formula or a future value table. It has only been within the past 25 years that financial calculators have become an inexpensive and widely used tool for computing time-value-of-money outputs. The technological advancement associated with modern software and financial calculators has made computing future values easy and quick. Because of the efficiency of modern software, it is possible that some financial planners may have forgotten the basic underlying formulas and methodology of certain calculations. The purpose of this brief paper is to provide a refresher of the three most common types of future value calculations used by financial planners with special emphasis given to the application of the geometrically varying annuity formula.

The Future Value Formula

The simplest future value calculation involves solving for an unknown value using a present value and a known interest rate and compounding period. When first learning to calculate the future value of a sum, nearl+-y all financial planners were instructed to use the following formula:

$$FV = PV(1 + i)^n$$

Where

PV = Present Value

i = Periodic Interest Rate

n = Number of Periods

This formula can be used to determine the future value of an asset.

Another formula is needed to solve for the future value of an annuity. The annuity in this case is a fixed payment over a certain number of periods. The future value of an annuity formula is as follows:

$$FV = p\left[\frac{(1+i)^n - 1}{i}\right]$$

Where

FV = Future Value

p = Periodic Payment

i = Periodic Interest Rate

n = Number of Payments

Solving for a future value of a fixed annuity using this formula is straightforward. For example, assume that Terry saves $3,000 a year into a Roth IRA. How much will Terry have accumulated in the account at the end of ten years if he can earn eight percent compounded annually? Solving this problem involves replacing p with $3,000, i with 8 percent and n with 10, as follows:

$$FV = \$3{,}000\left[\frac{(1 + .08)^{10} - 1}{0.08}\right]$$

$$= \$3{,}000(14.4866) = \$43{,}459.69$$

In today's world of computer programs and sophisticated calculations, it is unlikely that many planners regularly use either the future value formula or the future-value-of-an-annuity formula. While it is true that college professors often require students taking classes in personal financial planning to solve problems using these formulas, few financial planners do. When working directly with a client, most, if not all, financial services professionals solve future value problems using a spreadsheet, software program or financial calculator. Solving a future-value-of-an-annuity problem using a financial calculator, such as the Texas Instruments BAII Plus, for instance, is easy. An advisor would enter 3000 PMT, 8 I/Y, 10 N and CPT (that is, compute) FV. When rounded, the result is $43,460.00. Common types of future value calculations are not complicated to solve.

The Geometrically Varying Annuity

There is a future value calculation, however, that cannot be solved as easily using the time-value-of-money functions on a financial calculator. A special form of a future-value-of-an-annuity problem is a *geometrically varying annuity*. In essence, the geometrically varying annuity problem exists when the payment is assumed to increase on a per-period basis. The two most practical ways to solve a geometrically varying annuity problem include creating a spreadsheet or using a formula. The traditional future value of an annuity formula must be adapted to meet this special assumption. The following example, continued from the problem presented above, provides a review of the geometrically varying annuity concept.

Assume that Terry still plans to put $3,000 initially into a Roth IRA this year. He can earn eight percent on an annualized basis. His time horizon is ten years, but now it is learned that Terry will increase his annual contribution by three percent in each subsequent year. Using the traditional time-value-of-money formula to solve this problem will likely result in frustration. Neither the formula nor financial functions on most calculators can easily accommodate a growing payment.[1]

Fortunately, there is a simple formula that can be used to solve for the future value of a geometrically varying annuity. As with all time-value-of-money approaches, the following formula can be inserted without much difficulty into a needs analysis spreadsheet as needed.

$$FV = p\left[\frac{(1 + i)^n - (1 + k)^n}{i - k}\right]$$

Where

FV = Future Value

p = Periodic Payment

i = Interest Rate

k = Growth Rate of Payment

n = Number of Payments

Solving the problem is straightforward using the formula as follows:

$$FV = \$3{,}000 \left[(1 + .08)^{10} - (1 + .03)^{10} \right]$$

$$.08 - .03$$

$$= \$3{,}000(16.30) = \$48{,}900$$

In this case, the fact that Terry's initial deposit into the IRA grows by three percent annually means that he will have approximately $5,400 more in the account at the end of ten years than if he had used a traditional annuity payment savings strategy. Growing payments over a long-term planning horizon can make a significant impact on meeting a client's financial goals.

Summary

As suggested at the outset of this paper, future-value problems and calculations are among the core concepts underlying the financial planning process. Whether computed with a formula, calculator or software, the ability to accurately determine a future value is a valuable financial planning skill. While it is true that many times future value calculations are computed internally within software, there are times when having access to the actual formulas used can be beneficial. As such, this paper has served as a brief review of the traditional future value formula, the future-value-of-an-annuity formula and the geometrically varying annuity formula, which are of the utmost importance to financial planners.

Endnote

1. It should be noted, however, that the uneven cash-flow method, as a common time-value-of-money calculation, may be used to solve this type of problem, but the process is cumbersome. It is also possible to solve the problem using an HP-12C or TI-BA-II Plus calculator. This process requires that the present value of the initial payment be computed using an annuity due and serial interest rate. The result then needs to be stored and subsequently used in a traditional future-value calculation. Some financial planners consider the calculator method to be conceptually difficult and awkward.